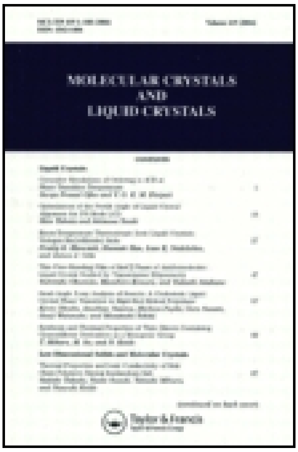

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Influence of a Magnetic Field on the Periodic Splay-Stripes in Hybrid Aligned Nematics

Amelia Sparavigna [a] , Lachezar Komitov [b] & Alfredo Strigazzi [b]
[a] Dipartimento di Fisica, INFM, Politecnico di Torino, 1-10129, Torino, Italy
[b] Physics Department, Chalmers University of Technology, S-41296, Göteborg, Sweden
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INFLUENCE OF A MAGNETIC FIELD ON THE PERIODIC SPLAY-STRIPES IN HYBRID ALIGNED NEMATICS

AMELIA SPARAVIGNA°, LACHEZAR KOMITOV*, and ALFREDO STRIGAZZI°*

°Dipartimento di Fisica and INFM, Politecnico di Torino, I-10129 Torino, Italy
*Physics Department, Chalmers University of Technology, S-41296 Göteborg, Sweden

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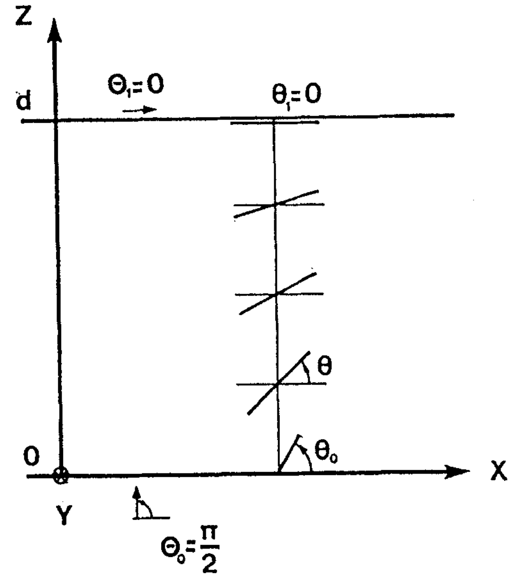
Abstract An aperiodic hybrid alignement can only appear above a critical thickness in a nematic layer with weak anchoring. Here the influence on the critical thickness of a magnetic field normal to the cell plates is reported. Moreover, the existence of periodic solutions, previously found in a hybrid cell without external field, is discussed in the presence of the magnetic field and in the case of strong anchoring at the wall, where the easy direction is unidirectional planar.

Keywords: hybrid, nematics, alignment, magnetic field

INTRODUCTION

It is well known that the action of a magnetic field on a nematic liquid crystal (NLC) layer uniformly oriented may produce a second-order-phase transition (the so-called Fréedericksz transition), which forces the material to change its configuration to a distorted one. In a cell with the same planar (P-) or homeotropic (H-) anchoring at both walls, the destabilizing field H must be perpendicular- or parallel- to the walls, respectively, the susceptivity anisotropy χ_a being positive.

On the contrary, if the NLC cell has opposite boundary conditions (P-alignment at one of the walls, H-alignment at the other one), the director configuration can continuously vary from one side to the other: this is the so-called hybrid aligned nematic (HAN) cell. In the presence of a magnetic field normal to the cell plates, the director profile, i.e. the average direction of the molecular long axes, simply changes locally, according to the intensity of H, without any threshold[1,2]. Furthermore, a HAN layer in the absence of external field exhibits a distortion similar to the one which characterizes the a)



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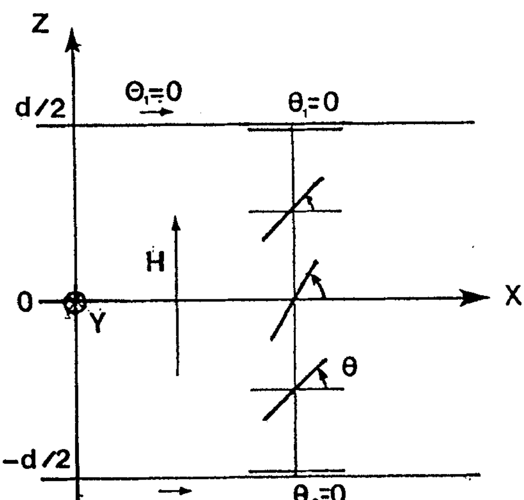

FIGURE 1. a) Typical aperiodic director profile in a HAN cell with strong planar anchoring at the wall z = d and weak homeotropic anchoring at the wall z = 0. Θ_1 and Θ_0 are the tilt angles characterizing the easy directions on the plates, due to the surface treatment. b) Unidirectional planar cell of thickness d, strongly anchored at both sides, subjected to a magnetic field **H** inducing an aperiodic Fréedericksz transition. Note the similarity of the two cases a), b).

director configuration in one half of a previously uniformly aligned cell subjected to the Fréedericksz transition: note that in the first situation the layer thickness plays the same role as the magnetic field in the latter one. In figure 1 a) a HAN cell is shown, with strong anchoring[3] at one of the walls, where the easy direction is planar. Figure 1 b) illustrates the analogy with a P-cell strongly anchored, undergoing a Fréedericksz transition due to a magnetic field **H**.

On the other hand, in 1985 Lonberg and Meyer[4] found the existence of a new kind of spatially periodic Fréedericksz transition for a class of polymer nematics composed of very long molecules. If such a NLC is arranged in a P-cell subjected to a magnetic field, above a certain threshold the invariance of the translation is broken along the axis normal to the undistorted director in the boundary plane: the equilibrium configuration of the director in the sample assumes now a static spatial periodicity. In ref. /4/, the authors proposed, as explanation of the occurrence of the periodic distortion, the fact that, in NLC composed by very long molecules, the elastic constant associated with splay (K_{11}) is much larger than the one associated with twist (K_{22}): thus a mixed distortion, consisting of a twist superimposed to a small splay is more favored than a simple but pronounced splay. Lonberg and Meyer deduced by means of a numerical calculation the critical ratio $r = K_{22}/K_{11}$ for the periodic Fréedericksz transition in a cell with strong anchoring: this critical value ($\sim$0.3) was subsequently confirmed through an analytical approach[5]. In refs. /5-12/ the effect of weak anchoring and or of different external fields on the transition to the splay-stripes in a P-cell is described.

Could a static periodic pattern appear in a HAN cell without external field? First of all, it must be pointed out that an aperiodic HAN cell with finite anchoring energy can be stable only if the cell thickness d is greater than a threshold d_a [13, 14]. Obviously, d_a vanishes in the case of strong anchoring at both walls. In fact, also a periodic hybrid alignment configuration (PHAN) is possible, where the leading parameter is the cell thickness. The phenomenon presents a threshold $d_p < d_a$. Close to the threshold d_p the periodic deformation is mainly a splay superimposed to a twist.

The first experimental data were reported by Lavrentovich and Pergamenshchik[15], who discussed by means of a numerical approach the behavior of the threshold in the case of no anchoring for twist.

Recently[11] the occurrence of PHAN has been theoretically investigated for a cell with arbitrary weak tilt and twist anchoring at both walls, but with tilt anchoring strength[17, 18] greater at the one of the walls, where the easy direction is unidirectional planar. Here the planar alignment appears to be the preferred one for the layer thickness d smaller than the threshold value d_p. Hence the occurrence of splay-stripes was obtained by means of linear analysis for a layer thickness in the interval $d_p < d < d_a$. In ref. /16/ the dependence of d_p on the twist-splay elastic ratio r and on the anchoring conditions is shown. Moreover, in ref. /19/ the case of r=1 is analysed, and the possibility of PHAN configuration is reported, as a function of the anchoring conditions. Also the influence of the saddle-splay[20] elastic constant K_{24} on the PHAN threshold is discussed.

In the present paper we consider a NLC cell with opposite boundary conditions, strongly anchored at the P-wall and subjected to a magnetic field, in order to investigate the behavior of the periodic configuration as a function of the field and of the sample parameters (thickness, elastic ratio, and anchoring energies). The threshold thickness in the presence of the external field has been calculated and the competition between both field and cell thickness is discussed.

THEORY

Constitutive equations

Let us consider a nematic layer between two confining surfaces, z=0 and z=d, where z is the co-ordinate perpendicular to the walls. The easy axes are chosen in order to give H-alignment at the wall z=0, and P-alignment at z=d. The local orientation of the director **n** is determined by the tilt angle θ and by the twist angle φ, which are equal to zero when **n** is parallel to the x-axis (see figure 2). A magnetic field **H** is assumed to be parallel to the z-axis.

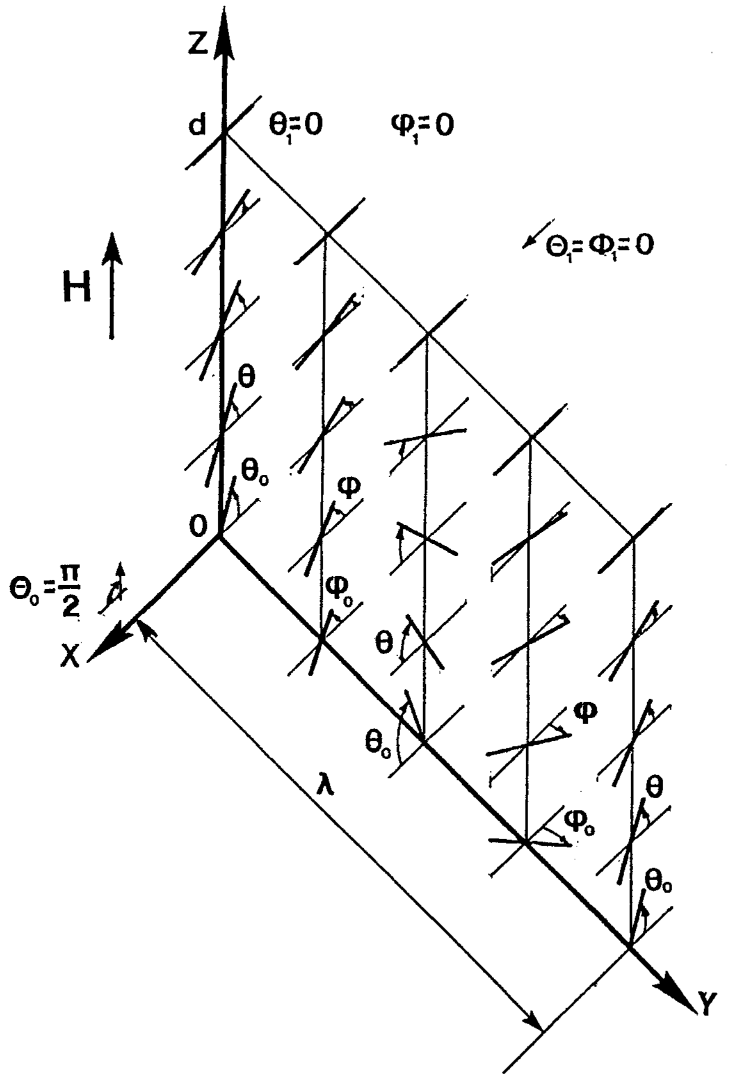

FIGURE 2. Director profile corresponding to the periodic splay-twist deformation (PHAN) occurring in a nematic layer with opposite boundary conditions, in the presence of a magnetic field **H** normal to the cell plates. The anchoring is weak at the lower wall z = 0, where the easy direction is characterized by $\Theta_0 = \pi/2$ (homeotropic alignment), while it is strong at the upper wall z = d, where the easy direction is given by $\Theta_1 = 0, \Phi_1 = 0$ (unidirectional planar alignment). λ is the spatial wavelenght of the in-plane distortion.

We are looking for transverse periodicity along the y-axis: hence θ and φ are dependent on y and z. With the aim of considering the effect of both **H** and d on the transition to splay-stripes, for the sake of simplicity we assume the anchoring at the P-wall to be strong. No restrictions are given on the tilt- and twist-anchoring strengths at the H-wall ($w_{\theta o}$ and $w_{\varphi o}$, respectively).

Since the goal of the present work is to obtain the lower threshold for the occurrence of the splayed PHAN-pattern, it is convenient to linearize the distortion close to the threshold itself, which separates the unidirectional planar undistorted configuration from the periodically deformed one. Thus the reduced free energy G of a cell unit, i.e. of a NLC box with dimensions d_x, λ, d along the x, y, z-axes respectively, is given by:

$$G = \int_0^{\lambda} dy \int_0^{d} dz \left\{ (\varphi_y + \theta_z)^2 + r\,(\theta_y - \varphi_z)^2 - h^2\,\theta^2 \right\} + \int_0^{\lambda} dy \left[r\,L_{\varphi o}{}^{-1}\,\varphi_o{}^2 - L_{\theta o}{}^{-1}\,\theta_o{}^2 \right] \tag{1}$$

where λ is the periodic pattern wavelength, $r = K_{22}/K_{11}$ is the twist-splay elastic ratio, $L_{io} = K_{jj}/w_{io}$ are de Gennes-Klêman extrapolation lengths[21, 22] for tilt- and for twist-anchoring at the H-wall ($i=\theta,\varphi$ and $j=1, 2$, respectively), and $h = H\,(\chi_a/K_{11})^{\frac{1}{2}}$ is the reduced magnetic field, having the meaning of the inverse of the magnetic coherence length[5]. For the sake of simplicity, the possible effect of the saddle-splay elastic constant is neglected, assuming $K_{24} = -K_{22}$.

The usual variational approach[23] provides the linearized Euler-Lagrange (EL) bulk equations:

$$\begin{cases} r\,\theta_{yy} + \theta_{zz} + (1-r)\,\varphi_{yz} + h^2\,\theta = 0 \\ \varphi_{yy} + r\,\varphi_{zz} + (1-r)\,\theta_{yz} = 0 \end{cases} \tag{2}$$

with the linearized boundary conditions:

$$\begin{cases} \theta_{yo} + L_{\varphi o}^{\ -1}\varphi_o - \varphi_{zo} = 0 \\ \varphi_{yo} + L_{\theta o}^{\ -1}\ \theta_o + \theta_{zo} = 0 \\ \varphi_1 = 0 \\ \theta_1 = 0 \end{cases} \tag{3}$$

which are explicitly independent of r.

Aperiodic solution

The usual HAN profile does not exhibit twist: consequently the tilt angle is just given as a function O(z). Hence the only EL equation reads

$$\theta_{zz} + h^2\ \theta = 0 \tag{4}$$

with harmonic solution

$$\theta = a\cos(hz) + b\sin(hz) \tag{5}$$

and boundary conditions

$$\begin{cases} \theta_{zo} + L_{\theta o}^{\ -1}\theta = 0 \\ \theta_1 = 0 \end{cases} \tag{6}$$

By inserting (5) in (6), a homogeneous system in the integration constant a, b is obtained:

$$\begin{cases} L_{\theta o}^{\ -1}a + h\ b = 0 \\ a\cos(hd) + b\sin(hd) = 0 \end{cases} \tag{7}$$

which has a nontrivial solution for

$$\tan(hd) = h\, L_{\theta o} \tag{8}$$

Relation (8) is the form assumed by the Rapini-Papoular equation[24, 25] in the present case, giving implicitly the aperiodic threshold thickness[14] $d = d_a(h) < d_a(0) = L_{\theta o}$ (see figure 3).

By calculating the reduced free energy G of the cell unit for $d \gtrsim d_a$, close to the threshold, we recognize that the relevant G (d) is a minimum: thus the aperiodic solution is shown to be stable.

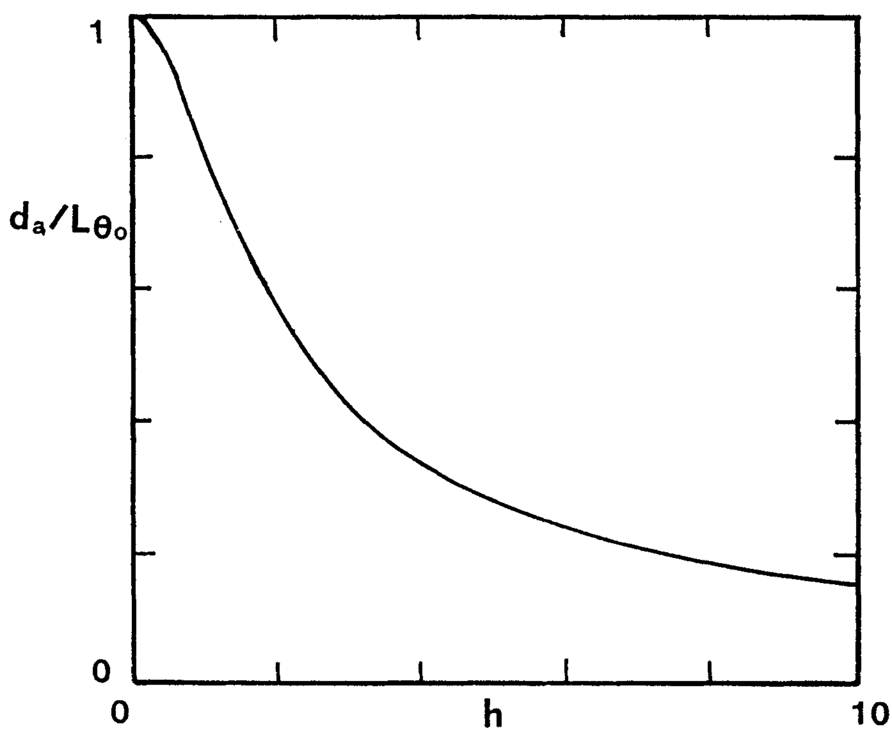

FIGURE 3. Threshold reduced thickness $d_a/L_{\theta o}$ for the aperiodic HAN distortion as a function of the reduced field h. When h increases, then $d_a/L_{\theta o}$ diminishes as well.

Periodic solutions

When $d < d_a(h)$, a direct transition to periodic splay-stripes with y-wave number $\beta = 2\pi/\lambda$ are possible: hence, according to the usual procedure[5, 16], the real part of the z-wave number of both tilt- and twist angles θ (y,z) and φ (y,z) is given by k, q satisfying the relations

$$\begin{cases} \theta = \left[a_1 \cos kz + b_1 \sin kz + c_1 \operatorname{ch} qz + d_1 \operatorname{sh} qz\right] \cos \beta y \\ \varphi = \left[A_1 \cos kz + B_1 \sin kz + C_1 \operatorname{ch} qz + D_1 \operatorname{sh} qz\right] \sin \beta y \end{cases} \tag{9}$$

with the constraints, obtained by inserting (9) into EL equations:

$$\begin{cases} k^2 = (h^2/2)\left\{1 - 2\beta^2/h^2 + \left[1 + 4\beta^2 (1-r)/(r h^2)\right]^{\frac{1}{2}}\right\} > 0 \\ q^2 = (h^2/2)\left\{-1 + 2\beta^2/h^2 + \left[1 + 4\beta^2 (1-r)/(r h^2)\right]^{\frac{1}{2}}\right\} > 0 \end{cases} \tag{10}$$

The first inequality of system (10) is satisfied only for a sufficiently high value of the reduced field h: otherwise we have $m = ik$, with

$$m^2 = (h^2/2)\left\{-1 + 2\beta^2/h^2 - \left[1 + 4\beta^2 (1-r)/(r h^2)\right]^{\frac{1}{2}}\right\} > 0 \tag{11}$$

and the solutions are of the form

$$\begin{aligned} \theta &= \left[a_1 \operatorname{ch} mz + b_1 \operatorname{sh} mz + c_1 \operatorname{ch} qz + d_1 \operatorname{sh} qz\right] \cos \beta y \\ \varphi &= \left[A_1 \operatorname{ch} mz + B_1 \operatorname{sh} mz + C_1 \operatorname{ch} qz + D_1 \operatorname{sh} qz\right] \sin \beta y \end{aligned} \tag{12}$$

In both cases the solutions must satisfy the EL system (2): hence the wave numbers of the director modulation along z- and y-axis are connected together and to both field and elastic ratio by the links

$$\begin{cases} R = (1-r)\, k\beta/(\beta^2 + rk^2) = (r\beta^2 + k^2 - rh^2)/\left[k\beta\,(1-r)\right] \\ T = (1-r)\, q\beta/(rq^2 - \beta^2) = (r\beta^2 + q^2 - rh^2)/\left[q\beta\,(1-r)\right] \end{cases} \tag{13}$$

and just four integration constants are linearly independent:

$$\begin{cases} a_1 = B_1/R \\ b_1 = -A_1/R \\ c_1 = D_1/T \\ d_1 = C_1/T \end{cases} \tag{14}$$

By inserting the solutions (9), (12) into the boundary conditions (3), and taking into account the bulk connections (13), (14), a homogeneous system is obtained again in the four independent integration constants. This fact implys the vanishing of the 4^2-determinant of coefficients D, which is given by:

$$D = \begin{vmatrix} L_{\varphi_0}(\beta - kR) & -L_{\varphi_0}(\beta/T + q) & R & 1 \\ -1 & 1/T & L_{\theta_0}(\beta R + k) & L_{\theta_0}(q/T + \beta) \\ R \sin kd & \mathrm{sh}\, qd & R \cos kd & \mathrm{ch}\, qd \\ -\cos kd & (1/T)\, \mathrm{ch}\, qd & \sin kd & (1/T)\, \mathrm{sh}\, qd \end{vmatrix} \tag{15}$$

Let us point out that when $h \to 0$, thus k becomes imaginary ($k=im$ with m real number), and from (13) R becomes an imaginary number too. This means that D keeps the same form described in (15), all determinant elements being real: simply, $\sin kd = i\ \mathrm{sh}\ md$, and $\cos kd = \mathrm{ch}\ md$, according to (11).

Hence the relation $D = 0$ is the generalized Rapini-Papoular equation, providing the threshold d_p for the transition to the splay-stripes.

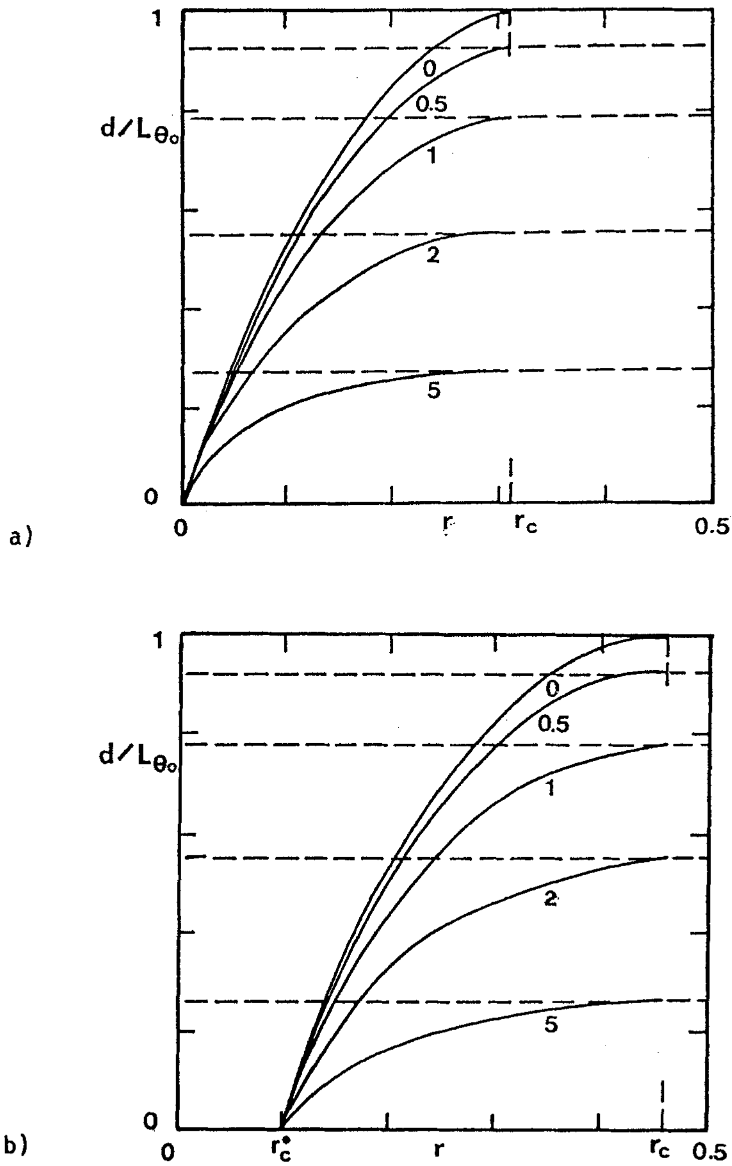

FIGURE 4. Threshold reduced thickness $d_p/L_{\theta o}$ for the occurence of the PHAN pattern as a function of the twist-splay elastic ratio r, for different values of the reduced field h. In a) the anchoring ratio $L_{\varphi o}/L_{\theta o} = 0$, and the critical value r_c hindering PHAN is 0.31. In B) the anchoring ratio turns out to be $L_{\varphi o}/L_{\theta o} = 0.1$, $r_c \simeq 0.46$ and another critical value $r^*_c \simeq L_{\varphi o}/L_{\theta o}$ hindering P-alignment is shown. For a given reduced field h, PHAN is allowed in the region above the continuous line and below the dashed line, describing the HAN-threshold $d_a(h)/L_{\theta o}$.

PHAN threshold

The Rapini-Papoular generalized equation $D = 0$ gives the implicit function $d = d(\beta;r,h,L_{\varphi o}/L_{\theta o})$. By computing numerically d vs. β for fixed convenient values of h and $L_{\varphi o}/L_{\theta o}$ the minimum of such a function is obtained, providing the PHAN threshold $d_p(\beta_p;r,h,L_{\varphi o}/L_{\theta o})$. The relevant wave number β_p characterizes the splay-stripes periodicity at the threshold.

Figures 4 a), b) show the behavior of $d_p/L_{\theta o}$ as a function of the elastic ratio r for different reduced external field h when $L_{\varphi o}/L_{\theta o}$ = 0, 0.1, respectively. For each curve, PHAN is allowed in the region above the considered curve (continuous line) and below the value $d_a(h)/L_{\theta o}$ (dashed line); whereas below the curve $d_p/L_{\theta o}$ only the uniform P-alignement takes place in the whole cell.

Note that a critical value r_c of the elastic ratio hindering PHAN appears, practically independent of h and dependent only on $L_{\varphi o}/L_{\theta o}$: in figure 4 a), $r_c \cong 0.31$, like in the P-cell considered in ref. /4/. There can exist also another critical point r^*_c, hindering the P-undeformed structure: when $r < r^*_c$, thus the PHAN deformation is favored even if $d \rightarrow 0$. Such a critical value turns out to be practically equal to the anchoring ratio $L_{\varphi o}/L_{\theta o}$.

In figure 5 the behavior of $d_p/L_{\theta o}$ as a function of the reduced field h is reported, for different values of the elastic ratio $r < r_c \cong 0.31$, when $L_{\varphi o}/L_{\theta o} = 0$, figure 5 a), and $r \rightarrow r_c \cong 0.46$ when $L_{\varphi o}/L_{\theta o} = 0.1$, figure 5 b), respectively. Note that for $r \rightarrow r_c$ the curve $d_p(h)/L_{\theta o}$ collapses onto the function $d_a(h)/L_{\theta o}$ (line a).

On the other hand, by calculating the reduced free energy G of the NLC layer for $d \gtrsim d_p$ close to the threshold, the minimum condition of G is recognized, ensuring us that the found PHAN-solution is stable.

CONCLUSION

In this paper the conditions for transition to static periodic distortions were analyzed, in a NLC cell with opposite boundary constraints, with strong anchoring at the P-wall, in the presence of a magnetic field normal to the cell plates. The magnetic field was found to favor both PHAN and HAN distortions, diminishing the relevant a)

b)

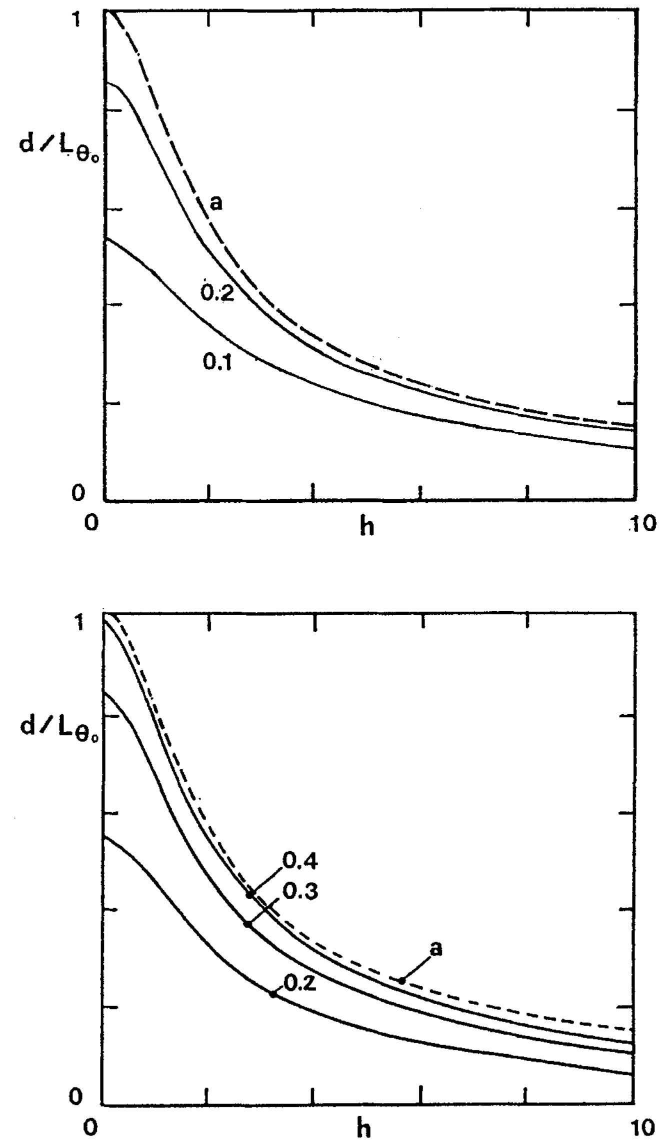

FIGURE 5. Threshold reduced thickness $d_p/L_{\theta o}$ as a function of the reduced field h, for different values of the twist-splay elastic ratio r. In **a)** the anchoring ratio $L_{\varphi o}/L_{\theta o} = 0$, and the curves r = 0.1, 0.2 are reported. If $r \to r_c = 0.31$, then $d_p/L_{\theta o}$ collapses onto the limit $d_a/L_{\theta o}$ (dashed line a). In **b)** the anchoring ratio $L_{\varphi o}/L_{\theta o} = 0.1$, and the curves r = 0.2, 0.3, 0.4 are reported. If $r \to r_c = 0.46$, then $d_p/L_{\theta o}$ goes to the same limit, whereas if $r \to r^*_c = L_{\varphi o}/L_{\theta o}$, then $d_p/L_{\theta o}$ collapses onto the h-axis. For a given elastic ratio r, PHAN is allowed in the region above the continuous line and below the dashed line.

thresholds d_p and d_a. At fixed magnetic field, d_p decreases and the anchoring ratio $L_{\varphi_0}/L_{\theta_0}$ increases as well. The distortions are found to be stable. The PHAN deformation is forbidden at elastic ratios greater than a critical point r_c strongly dependent on the anchoring ratio $L_{\varphi_0}/L_{\theta_0}$, whereas the P alignment is forbidden for elastic ratios lower than another critical point r^*_c almost equal to $L_{\varphi_0}/L_{\theta_0}$.

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